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The Partial Donor Cell Method

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The Partial Donor Cell Method (PDM) as described here mixes a second-order scheme with the donor cell scheme. It keeps transported quantities monotone and minimizes diffusion. The PDM scheme in conjunction with a central difference scheme for the numerical solution of the hydrodynamic and magnetohydrodynamic equation has been used for a variety of problems. This scheme has been proven to be quite robust and can resolve contact discontinuities quite well. Several results of numerical simulations are presented.

I. INTRODUCTION

Hydrodynamic codes have been used to simulate physical events ever since the occurrence of computers. Although there has been considerable progress in numerical methods, the several major difficulties still remain. They are:

(1) transporting of quantities across the mesh,

(2) handling of shocks and contact discontinuities, which are inherent in compressible hydrodynamics, and,

(3) the restriction of the timestep by the Courant condition.

In general, a numerical code should be at least of second-order in space and first order in time. It should be Galilean invariant to the extent possible. This means, for instance, the dissipation in a shock of a given Mach number should be independent of the shock velocity in a given mesh.

Codes should be conservative to the extent possible. There are the conservation laws for mass, momentum, and energy. Directly using total energy conservation, one obtains the internal energy, and hence the temperature, by subtracting the kinetic energy from the total energy. This may introduce large errors for the internal energy. Therefore it is not always possible to determine the temperature with the accuracy necessary to compute chemical reactions and radiation losses, especially in multidimensional problems. One is thereby forced to use the equation for the internal energy which is not conservative. However, there is also entropy which needs to be considered. The contention is made that a numerical code can be stable only if it either loses energy or increases entropy.

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The restriction on the timestep by the Courant condition can be overcome in two ways, namely by using an implicit scheme or by limiting the characteristic velocities. In the first case one may lose significant phase information; in the second case, the velocities by which disturbances travel are limited below their physical value.

As this issue of the *Journal* is dedicated to the memory of Keith V. Roberts, it seems appropriate to mention some of the early work. It was in 1960, when Keith was with the UKAEA in Harwell, that we simulated successfully the dynamic behaviour of a theta pinch (Ref. [1, Fig. 3]). Out of this came a whole series of numerical simulations of plasma pinches. I think it was then that Keith coined the term "computational physics."

This paper, which discusses a simple numerical scheme, is a refinement of the rather simplistic method we used in our early work to overcome the above-mentioned obstacles.

Section II discusses a simple transport scheme, namely the "partial donor cell method" (PDM). In general, a transport scheme has to be monotonic. It should not be diffusive. Furthermore, for zero velocities the transported quantities should not change. The PDM scheme as discussed mixes the donor cell method with a second-order scheme so as to minimize the diffusion.

Section III shows results of a one-dimensional transport and compares it with the "flux corrected transport" method. Also a two-dimensional example is given.

Section IV discusses the implementation of the PDM scheme into a hydrocode and gives the numerical formulae for solving the hydrodynamic equations.

Finally, Section V gives some results of numerical simulations and discusses the relative merits of a more complicated scheme, such as the "MUSCL" code versus the simple code presented here.

II. Transport Scheme

All modern transport schemes which involve a second or higher interpolation scheme consist of mixing this scheme with a lower order or first-order scheme in order to avoid the occurrence of extrema or, in other words, to be monotone.

The "partial donor cell method" (PDM) (Ref. [2]) discussed here mixes a second-order scheme with a donor cell method in such a way as to minimize diffusion and assure monotonicity. It has been designed to achieve this with a minimum of operations, therefore keeping the computer time for large three-dimensional hydrodynamic codes to a minimum.

Let us consider the continuity equation for a variable f with a velocity v in a cartesian mesh.

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial x}(vf). \tag{II.1}$$

First let us define some operators

$$\nabla^+ f_j = f_{j+1} - f_j, \qquad \nabla^- f_j = f_j - f_{j-1} \tag{II.2}$$

$$m^{\pm} f_j = \tfrac{1}{2}(f_j + f_{j\pm 1}) = f_j \pm \tfrac{1}{2}\nabla^{\pm} f_j \tag{II.3}$$

$$\nabla^2 = \nabla^+ \nabla^- = \nabla^+ - \nabla^- \tag{II.4}$$

In order to write the numerical scheme without indices, it is assumed that the operators act on j or $j+\frac{1}{2}$, respectively. Furthermore, a $\hat{}$ defines the variable at a time $t+dt$ or $t+\frac{3}{2}dt$, respectively.

We will consider two integration schemes, namely the central difference scheme and the Lax – Wendroff scheme. The partial donor cell method, PDM, will mix these second-order schemes with the first-order donor cell scheme in such a way that the diffusion is minimized and the monotonicity is achieved:

(a) The central difference scheme. In this scheme f is defined at integral points in space in time and v at half integral points in space in time. With the help of the above defined operators, this scheme can be written as

$$\hat{f} = f - dt\,\nabla^-(v m^+(f))/\nabla^-(m^+(x)). \tag{II.5}$$

(b) Lax–Wendroff. Here all quantities are defined at the same points in space and time. One defines an intermediate quantity at half points in space and time by

$$\bar{f} = m^+ f - \frac{dt}{2}\nabla^+(vf)/\nabla^+(x) \tag{II.6}$$

and, in the second step, the updated variable is given by

$$\hat{f} = f - dt\nabla^-(\bar{v}\bar{f})/\nabla^-(m^+(x)). \tag{II.7}$$

In the discussion here, it does not matter whether the velocity is updated or not.

It is well known that the cental difference scheme is unstable with an amplification factor A

$$A^2 = 1 + \frac{v^2\,dt^2}{dx^2} \tag{II.8}$$

In contrast, the Lax–Wendroff scheme has an additional diffusion term of

$$D = \tfrac{1}{2}v^2\,dt \tag{II.9}$$

which suppresses this instability. For a constant velocity it is a second-order interpolation scheme and results in nonmonotonicity for functions which are steeper than a parabola.

In order to simplify the discussions in this section, we assume a constant velocity and a constant mesh spacing. Let us then define

$$\varepsilon = \frac{v\,dt}{dx}. \tag{II.10}$$

The central difference scheme can then be written as

$$\hat{f} = f - \varepsilon \nabla^- (m^+(f)) \tag{II.11}$$

and the Lax–Wendroff scheme as a one-step scheme as

$$\hat{f} = f - \varepsilon \nabla^- \left(m^+ f - \frac{\varepsilon}{2} \nabla^+ f \right). \tag{II.12}$$

Let us introduce the partial donor cell method (PDM) by defining the PDM operator M as follows

$$M^\pm = m^\pm - \operatorname{sign}(v) \frac{\eta}{2} \nabla^\pm \tag{II.13}$$

for the central difference scheme and

$$M^\pm = m^\pm - \operatorname{sign}(v)(1 - |\varepsilon|) \frac{\eta}{2} \nabla^\pm \tag{II.14}$$

for all Lax–Wendroff scheme. The diffusion coefficients will be determined in such a way as to keep the scheme monotonic and to minimize diffusion. The FCT scheme by Boris (Ref. [3] does this in a different way by first introducing a large diffusion and then, in a second step, reducing the diffusion in such a way as not to introduce new extrema. As the first diffusion step is independent of the velocity, this may result in changing the function f for zero velocity.

In the paper by Hain *et al.* (Ref. [1]) which simulated the dynamics of pinches, we suppressed the occurrence of extrema during transport by restricting the transported value to lie between the original value and values at the corresponding upwind neighbor point. It can easily be shown that this scheme is almost conservative. In the early days we were just not sophisticated enough to worry about such details as exact conservation.

In formulae (II.13) and (II.14) one can easily see that $\eta = 1$ corresponds to the donor cell method. For the Lax–Wendroff scheme the maximal allowable diffusion is reduced by a factor of $1 - |\varepsilon|$. In order to simplify our discussion even further we will only consider the central difference scheme for the proof of monotonicity. The PDM scheme can then be written as

$$\hat{f} = f - \varepsilon \nabla^- (M^+ f). \tag{II.15}$$

Furthermore one can assume that $\nabla^- f > 0$. The proof for $\nabla^- f < 0$ goes in the same fashion. With this assumption the condition for monotonicity can be written as

$$0 < \varepsilon \nabla^- (M^+ f) < \nabla^- f. \tag{II.16}$$

Using the operator definitions from above, and leaving off the function f, this can be written as

$$-2\nabla^- < \nabla^- [(1-\eta)\nabla^+] < 2\left(\frac{1}{\varepsilon} - 1\right)\nabla^- \tag{II.17}$$

or

$$-2\nabla^- < (1-\eta)\nabla^+ - (1-\eta^-)\nabla^- < 2\left(\frac{1}{\varepsilon} - 1\right)\nabla^-. \tag{II.17a}$$

In the determination of the diffusion parameter, the direction in which the flow occurs is taken into account. In general, the PDM operator M^+ is determined by the formula

$$\begin{aligned} M^+ = m^+ &- \frac{1}{2}\operatorname{sign}(v)\cdot\operatorname{sign}(\nabla^+)\cdot\max[0, |\nabla^+| \\ &- \frac{A}{4}(1-\operatorname{sign}(v))\,|\operatorname{sign}(\nabla^+) + \operatorname{sign}(\nabla^{++})\,|\nabla^{++}| \\ &- \frac{A}{4}(1+\operatorname{sign}(v))\,|\operatorname{sign}(\nabla^- + \operatorname{sign}(\nabla^+))\,|\nabla^-|\,], \end{aligned} \tag{II.18}$$

where A is the parameter which determines the amount of diffusion. $A = 0$ will result in $\eta = 1$, which means the donor cell method.

From Eqs. (II.17) and (II.18) one can see that it is necessary to consider different cases for proving the monotonicity. Using the definition of the PDM operator for $v > 0$ and $\nabla^- > 0$ it is

$$\begin{aligned} \eta\nabla^+ &= \operatorname{sign}(\nabla^+)\max(0, |\nabla^+| - \frac{A}{2}(\operatorname{sign}(\nabla^+) + 1)\nabla^-) \\ \eta\nabla^- &= \operatorname{sign}(\nabla^-)\max(0, |\nabla^-| - \frac{A}{2}(\operatorname{sign}(\nabla^{--}) + 1)\,|\nabla^{--}) \end{aligned} \tag{II.19}$$

(∇^{--} is a ∇^- operator shifted one step to the left). This results, for the different cases, in the following:

$$\begin{aligned} &(1)\quad (1-\eta)\nabla^+ = 0 \qquad \text{for} \quad \nabla^+ \leqslant 0 \\ &(2)\quad (1-\eta)\nabla^+ = \nabla^+ \qquad \text{for} \quad 0 < \nabla^+ \leqslant A\nabla^- \\ &(3)\quad (1-\eta)\nabla^+ = A\nabla^- \qquad \text{for} \quad \nabla^+ > A\nabla^- \end{aligned} \tag{II.20}$$

and

$$\begin{aligned}
&\text{(a)} \quad (1-\eta^-)\nabla^- = 0 && \text{for} \quad \nabla^{--} \leqslant 0 \\
&\text{(b)} \quad (1-\eta^-)\nabla^{--} = A\nabla^{--} && \text{for} \quad 0 < \nabla^{--} \leqslant \frac{1}{A}\nabla^- \\
&\text{(c)} \quad (1-\eta^-) = \nabla^{--} && \text{for} \quad \nabla^{--} > \frac{1}{A}\nabla^-.
\end{aligned} \tag{II.21}$$

This can be combined in the following table:

$$-2\nabla^- < \begin{array}{c|ccc|}
 & 1 & 2 & 3 \\
\hline
a & 0 & \nabla^+ & A\nabla^- \\
b & -A\nabla^{--} & \nabla^+ - A\nabla^{--} & A(\nabla^- - \nabla^{--}) \\
c & -\nabla^- & \nabla^+ - \nabla^- & (A-1)\nabla^- \\
\hline
\end{array} < \left(\frac{2}{\varepsilon} - 1\right)\nabla^- . \tag{II.22}$$

In the table, $b1$ gives a condition for A, namely $A < 2$. The most stringent condition for ε is derived from $a3$. It gives

$$\varepsilon < \frac{2}{2+A} \tag{II.23}$$

For $A = 1$ this gives $\varepsilon <= \frac{2}{3}$ which is not too restrictive for hydrodynamics, which must also meet the Courant condition.

III. Numerical Results for Transport

First we show the results of a one-dimensional transport of a square wave in comparison with the partial donor cell method (Ref. [2]) and a FCT scheme (Ref. [3]). In Ref. [2], a more detailed comparison is given. Reference [2] also gives the effect of changing the parameter A (just to add a little confusion, it is called B in the reference). Increasing A to 2 gives results which are identical with FCT for all practical purposes.

The results for PDM obtained for $A = 1$ are given by the table in Fig. 1. The results are given for the transport of 10-point square wave transported 2 meshpoints and 20 meshpoints across with $\varepsilon = 0.2$, using the central difference scheme and the one-step Lax–Wendroff scheme. One sees that the PDM scheme is a little bit more diffusive than the FCT scheme, also the Lax–Wendroff scheme is more diffusive for both schemes. This is the result of the extra diffusion term for the Lax–Wendroff scheme as discussed in the previous chapter.

	t = 2.0				t = 20.0			
ANALYTIC	CENTRAL DIFF		LW		CENTRAL DIFF		LW	
VALUES	PDM	FCT	PDM	FCT	PDM	FCT	PDM	FCT
0 00	.000	000	000	.000	032	000	057	000
0 00	000	000	000	.000	077	000	118	.000
0 00	039	004	047	004	169	004	.222	.192
0 00	264	246	.289	293	.339	.436	.384	470
1.00	745	795	731	768	620	739	601	680
1 00	758	.964	943	.948	847	.889	783	823
1 00	996	.996	992	992	.942	956	893	911
1.00	1.000	1 000	999	1 000	.974	984	943	.960
1 00	1 000	1 000	1.000	1 000	980	995	.956	982
1.00	1 000	1 000	1 000	1 000	978	.998	955	988
1.00	1 000	1.000	1.000	1 000	964	.998	.935	.988
1 00	1 000	1.000	1.000	1.000	.924	998	.882	988
1 00	962	1 000	958	1.000	834	996	781	820
1 00	736	754	711	707	663	565	619	534
0 00	255	206	269	.232	382	261	401	.321
0.00	042	036	.057	.052	.159	111	220	178
0 00	005	.005	008	.008	062	.044	112	.091
0 00	000	.000	.000	000	.023	016	.053	.043

FIGURE 1

The graph in Fig. 2 shows the results of a two-dimensional transport. A square is rotated by 180°. It shows the results for a mesh of 201×201 points. Here $A = 0.8$ to add a bit more diffusion as is typically done in multi-dimensional hydrodynamic calculations. Although there is some erosion after turning 180°, the structure is still a essentially a square. The corners erode a bit faster, as one would expect. Compare this also with results obtained by S. Zalesak with his multi-dimensional FCT scheme (Ref. [8]).

IV. The Hydrodynamic Integration Scheme

In this section, we give the implementation of the partial donor cell method for the numerical solution of the hydrodynamic equations. This scheme discussed here is the simplest one which is second order in space and first order in time. It uses a central difference scheme in conjunction with the PDM method. As the central difference scheme is numerically unstable and the instabilities are suppressed by the PDM operator, it is able to retain contact discontinuities to a large extent. Shocks are handled by the von Neuman artificial viscosity technique in such a way that, for multi-dimensional problems, the shock width is a multiple of the mesh width in the different directions.

The hydrodynamic equations are solved using the density, the velocities, and the internal energy. For an ideal gas (gamma law) the pressure is used directly. It is not difficult to formulate these equations in a complete conservative way and then solve them numerically in this form. One of the reasons discussed in the introduction—namely to obtain the temperature as correctly as possible—requires that the equations be solved in the nonconservative way. Energy conservation provides a check on the correctness of the solution.

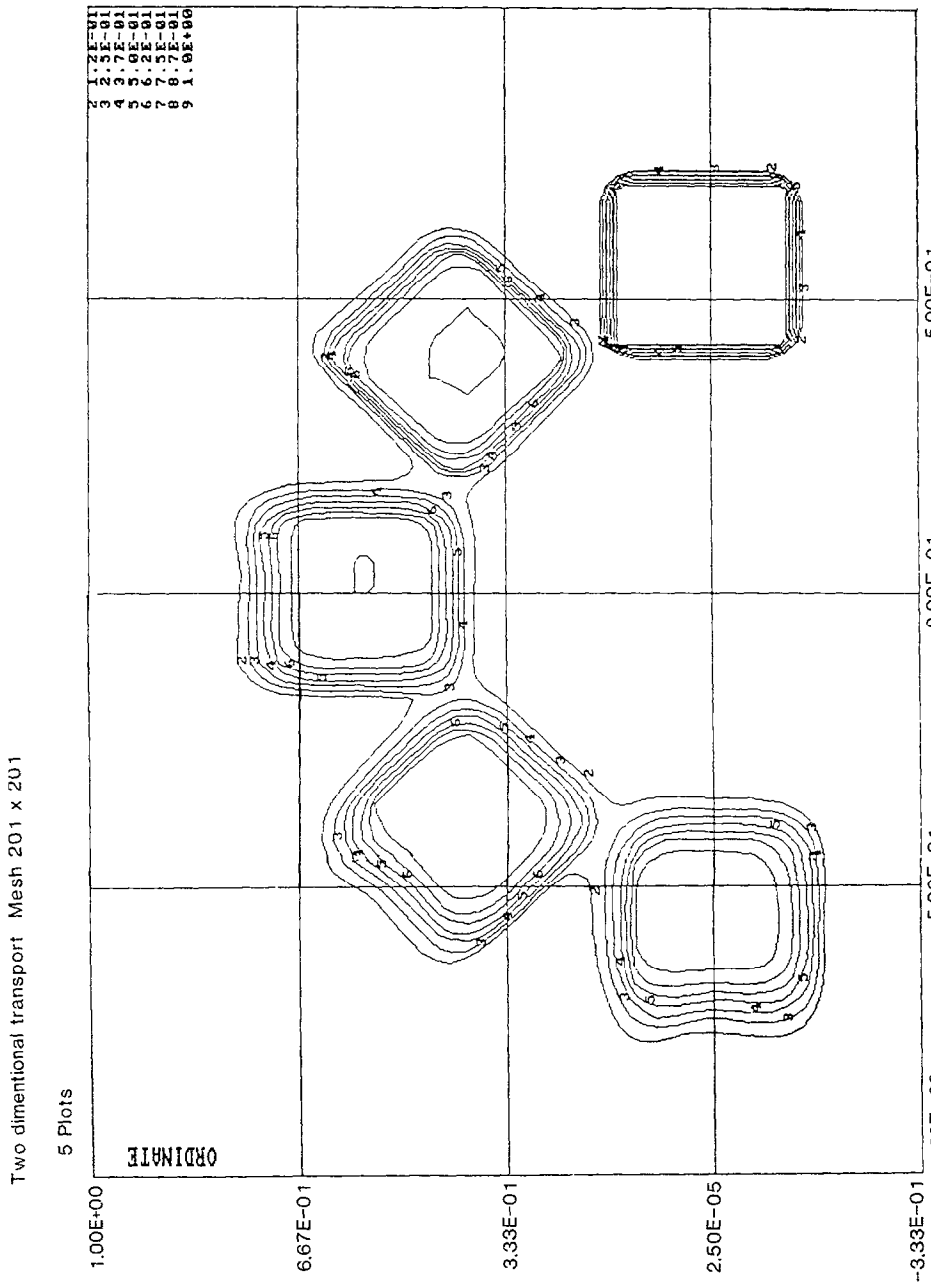
Two dimentional transport Mesh 201 x 201
5 Plots
ORDINATE
1.00E+00
6.67E-01
3.33E-01
2.50E-05
-3.33E-01
-1.00E+00
-5.00E-01
0.00E-01
5.00E-01
1 2.25E-01
2 2.25E-01
3 3.75E-01
4 3.75E-01
5 5.00E-01
6 6.25E-01
7 7.50E-01
8 8.75E-01
9 1.00E+00

FIGURE 2

In general, solving the equations in a conservative form does not guarantee a correct solution. (As an example, keep the density constant and then solve the rest in a conservative way.) In addition to conserving quantities, the right amount of entropy should be generated in the shocks and transported correctly. Test runs in 1D have shown that all quantities are handled correctly (with a relative error of less than 10**−4) except within the shock regions. These tests have also shown that in these regions one loses energy or increases entropy above its theoretical value. A conjecture is that a numerical scheme is stable only if it either loses energy or generates entropy above its theoretical value.

The hydrodynamic equations which are solved use the density ρ, the velocities v, and the pressure p. For simplicity a gamma law for pressure and internal energy is assumed using the continuity equation

$$\frac{\partial \rho}{\partial t} = -\sum_{\alpha} \nabla_{\alpha}(v_{\alpha}\rho) \tag{IV.1}$$

and the momentum equations

$$\frac{\partial v\beta}{\partial t} = -\sum_{\alpha} (v_{\alpha}\nabla_{\alpha})\, v_{\beta} - \frac{1}{\rho}\nabla_{\beta}(p + q_{\beta}). \tag{IV.2}$$

Here q_{β} is the pressure created by the artificial viscosity in the β direction. It is given by

$$q_{\beta} = -S_h \cdot \rho \cdot dx_{\beta} \min\left(0\cdot, \sum_{\alpha} \nabla_{\alpha}(v_{\alpha})\right)(c - dx_{\beta} \min(0, \nabla_{\beta} v_{\beta})), \tag{IV.3}$$

where c is the velocity of sound and dx_{β} is the mesh width in β direction.

The pressure equation is

$$\frac{\partial p}{\partial t} = -\sum_{\alpha} (\nabla_{\alpha}(v_{\alpha} p) + (\gamma - 1)(p + q_{\alpha})\nabla_{\alpha} v_{\alpha}). \tag{IV.4}$$

In the central difference integration scheme discussed here the density ρ and the pressure p (including the artificial shock pressure) are defined at integer points in space and time. The velocities v are defined at half integer points corresponding to their components and half integer points in time. As one can easily see, this allows one to compute the divergence of a flux in a simple way. The divergence then can be numerically written as

$$\operatorname{div}(vf) = \sum_{\alpha} \nabla_{\alpha}(v_{\alpha} f) = \frac{1}{\mathrm{Vol}} \sum_{\alpha} \nabla_{\alpha}^{-}(S_{\alpha} v_{\alpha} M_{\alpha}^{+}[f]), \tag{IV.5}$$

where S_{α} are the surface areas at the corresponding half grid points. M_{α} is the PDM operator in the alpha direction.

Note that this scheme and others which use a nonlinear diffusion operator are not suited for incompressible flows such as in wheather prediction. To show this we expand the divergence by the chain rule

$$\nabla(f \cdot g) = m(g)\,\nabla f + m(f)\,\nabla g \tag{IV.6}$$

which results in

$$\operatorname{div}(vf) = \frac{1}{\mathrm{Vol}} \sum_{\alpha} [\nabla_\alpha (S_\alpha v_\alpha)\, m_\alpha^- (M_\alpha^+ f) + \nabla_\alpha^- (M_\alpha^+ f)\, m_\alpha^- (S_\alpha v_\alpha)]. \tag{IV.7}$$

If numerically

$$\operatorname{div}(v) = \frac{1}{\mathrm{Vol}} \sum_{\alpha} \nabla_\alpha^- (S_\alpha v_\alpha) = 0 \tag{IV.8}$$

then

$$\operatorname{div}(vf) = \frac{1}{\mathrm{Vol}} \sum_{\alpha} [\nabla_\alpha^- (M_\alpha^+ f)\, m_\alpha^- (S_\alpha v) + \nabla_\alpha^- (S_\alpha v_\alpha)(m_\alpha^- M_\alpha^+ - 1) f], \tag{IV.9}$$

which is different from the pure transport equation. The use of Eq. (IV.5) can therefore lead to a numerical compression effect while the function f is perfectly conserved. In contrast, using pure transport of f—the first term in Eq. (IV.9)—will not be conservative if one uses a nonlinear diffusion operator in order to keep f monotonic.

The numerical scheme for solving the differential equation given in Eq. (IV.1), (IV.2), and (IV.3) can be written as follows.

The continuity equation is

$$\hat{\rho} = \rho - dt \frac{1}{\mathrm{Vol}} \sum_{\alpha} [\nabla_\alpha^- (S_\alpha v_\alpha M_\alpha^+ \rho)]. \tag{IV.10}$$

The pressure (energy) equation is

$$\hat{\rho} = \rho - dt \frac{1}{\mathrm{Vol}} \sum_{\alpha} \left[\nabla_\alpha^- (v_\alpha M_\alpha^+ p) + (\gamma - 1) \sum_{\alpha} (p + q_\alpha)\, \nabla_\alpha^- (S_\alpha v_\alpha) \right]. \tag{IV.11}$$

The momentum equations are

$$\hat{v}_\beta = v_\beta - dt \left[\sum_{\alpha} m_\alpha^+ m_\beta^-\, \nabla_\alpha^- (M_\alpha^+ v_\beta) / \nabla_\alpha^+ (x_\alpha) + \sum_{\sigma,\gamma} C_{\beta\alpha\gamma} v_\alpha v_\gamma \right.$$
$$\left. + \frac{1}{m_\beta^+ (\rho)} \nabla_\beta^+ (p + q_\beta) / \nabla_\beta^+ (x_\beta) \right]. \tag{IV.12}$$

The coefficients $c_{\alpha\beta\gamma}$ represent the coriolis and centrifugal terms. The shock pressure q_β is given by

$$q = -S_h\, dx_\beta \rho \min\left(0, \frac{1}{\mathrm{Vol}} \sum_\alpha \nabla_\alpha^- (S_\alpha v_\alpha)\right)$$
$$\times \left(C - \min(0, \nabla_\beta (S_\beta v_\beta)) \frac{dx_\beta}{\mathrm{Vol}}\right). \tag{IV.13}$$

The occurence of the difference of velocities in q may lead to difficulties in multi-dimensional problems. It has the advantage of steepening the shockfront. S_h is a constant. $S_h = \frac{1}{2}$ gives the same stability limit on the time step given by the resulting diffusion equation as the Courant condition. As test runs have shown, the best results are obtained by $S_h = \frac{1}{2}$.

Besides reflective and periodic boundary conditions which are imposed by the problem at hand, boundaries should be kept away so far as possible. The flow in and out of such boundaries should be diffusive in order to avoid the generation of waves.

V. Numerical Results

In the paper by Hain *et al.* [1], as mentioned in the Introduction, we computed the dynamics of a theta pinch according to the circuit data provided. We used an Eulerian mesh. The numerical scheme consisted of a Lagrangian compression step and then a second-order re-mapping) (see Section II). Shocks were handled with a von Neuman artificial viscosity. The Alfvén speed $B/\sqrt{4\pi\rho}$ was limited by not allowing the density to fall under a minimum value.

On invitation by Keith V. Roberts we went to England. Our work was greatly facilitated by the fact that Keith provided us with the newly released FORTRAN compiler and suggested that we write the program in the new high language. After completing the simulations, we compared the results with the experiment performed in Munich by Koeppendoerfer (Fig. 3). To our great satisfaction, they agreed remarkably well, thereby demonstrating the relevance of "computational physics."

The graph in Fig. 4 shows a simulation of Sod's (Ref. [4]) problem. The initial values are constant with a jump of pressure and density. The resulting Mach number is about 1.3. The theoretical solution of this Riemann problem can be computed and is given on the graph. It consists of a shockwave, a contact discontinuity, and a rarefaction wave, as shown in the graph. The numerical solution overshoots the density at the shock by a small amount. The contact discontinuity is represented quite well. If one defines the position of the shock as the point of the maximum negative divergence then this position agrees almost exactly with the theoretical value.

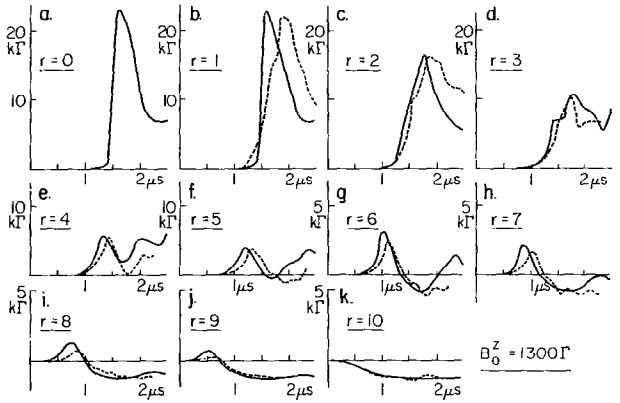

FIGURE 3

The graph in Fig. 5 shows a computation of a Mach 3 shock hitting a barrier. Results of different computation schemes have been discussed at length by Collela (Ref. [5]). The results obtained here were achieved without an extra procedure at the corner, as described in the paper by Collela. The presented results are in good agreement with Collela's results. But they lack a detailed structure. I think that the less complicated schemes like the PDM scheme, the FCT scheme and similar simple procedures are useful if one does not care about fine structures. The MUSCL code which uses a Riemann solver at every point may give better detailed results for the price of a complicated, and therefore relatively slow, code.

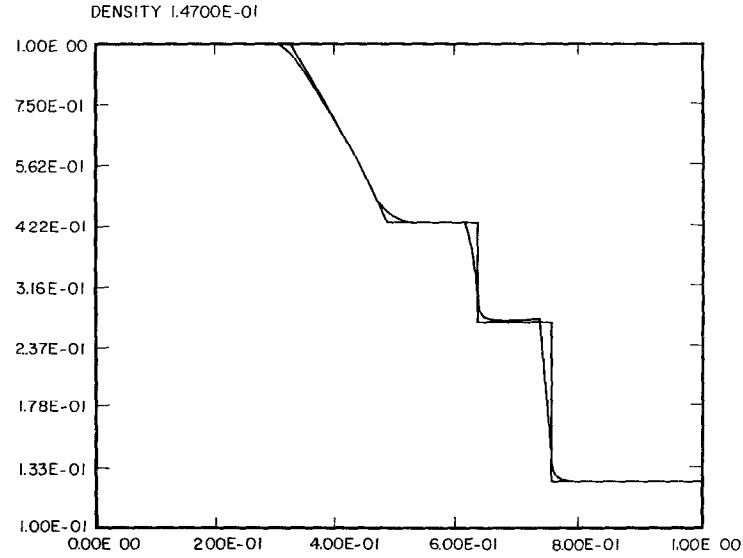

FIGURE 4

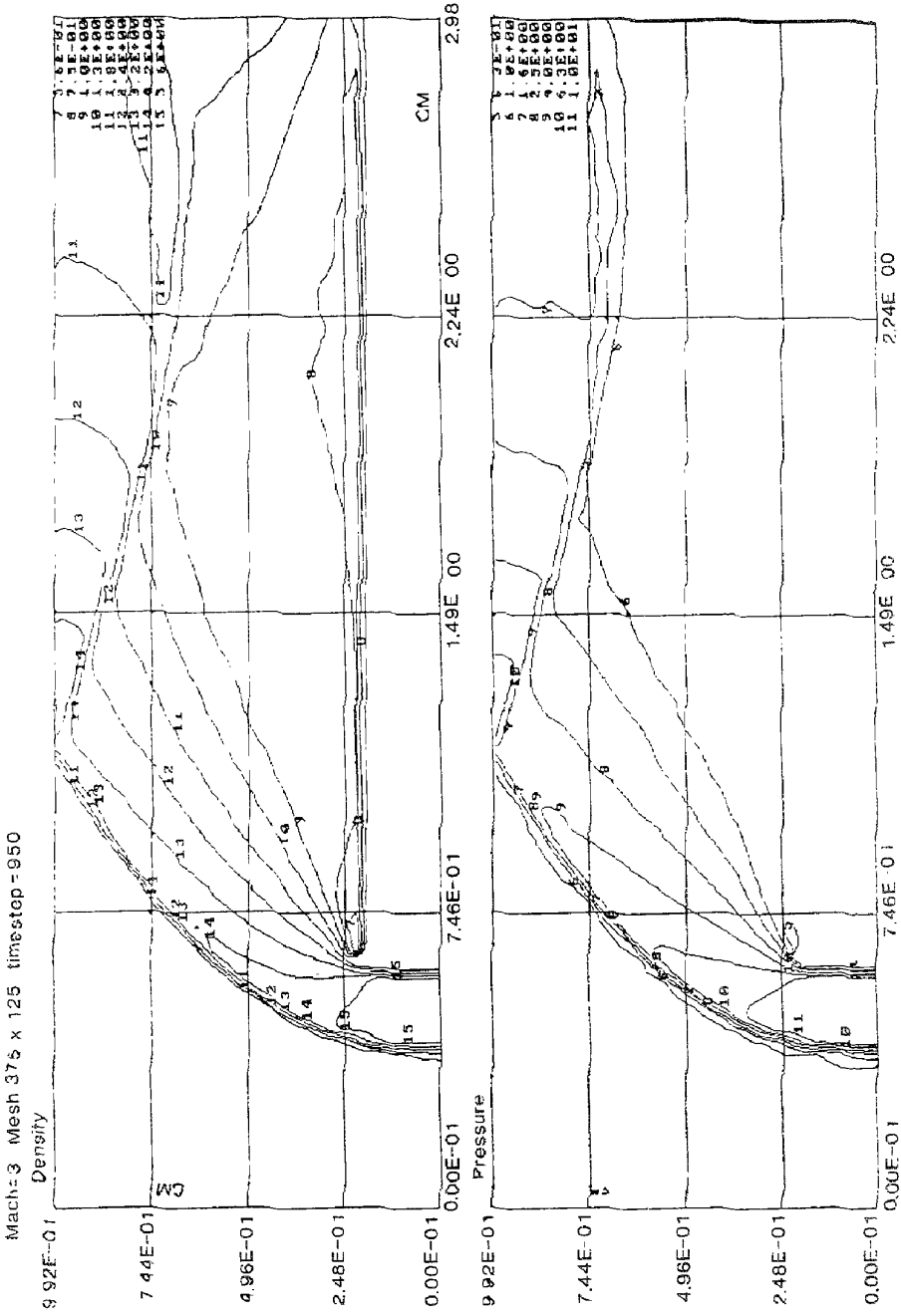
Mach=3 Mesh 375 x 125 timestep=950
Density
9.92E-01
7.44E-01
4.96E-01
2.48E-01
0.00E-01
0.00E-01
7.46E-01
1.49E 00
2.24E 00
2.98
CM
Pressure
9.92E-01
7.44E-01
4.96E-01
2.48E-01
0.00E-01
0.00E-01
7.46E-01
1.49E 00
2.24E 00

FIGURE 5

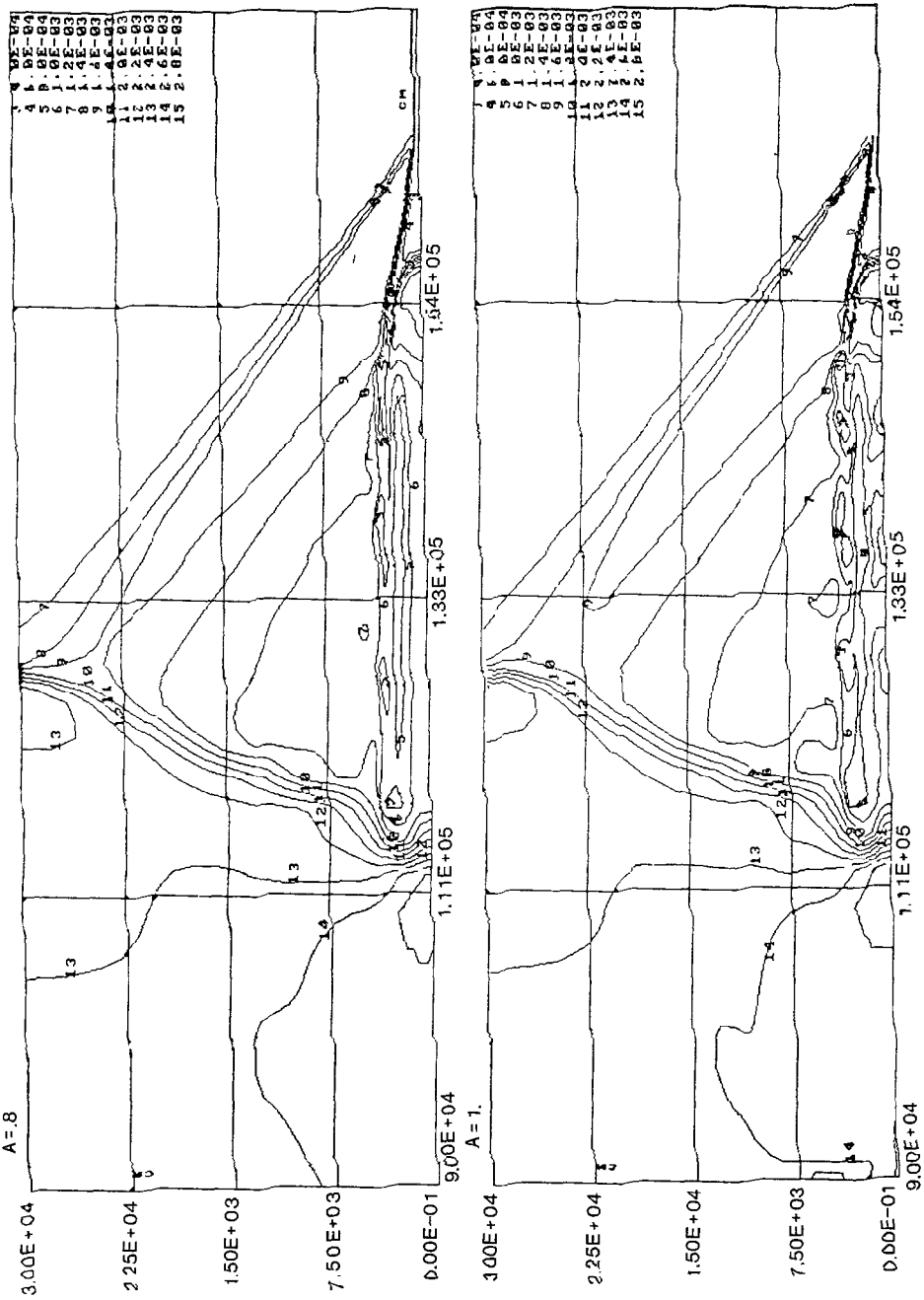
A=.8
A=1.
3.00E+04
2.25E+04
1.50E+03
7.50E+03
0.00E-01
9.00E+04
1.11E+05
1.33E+05
1.54E+05
cm

FIGURE 6

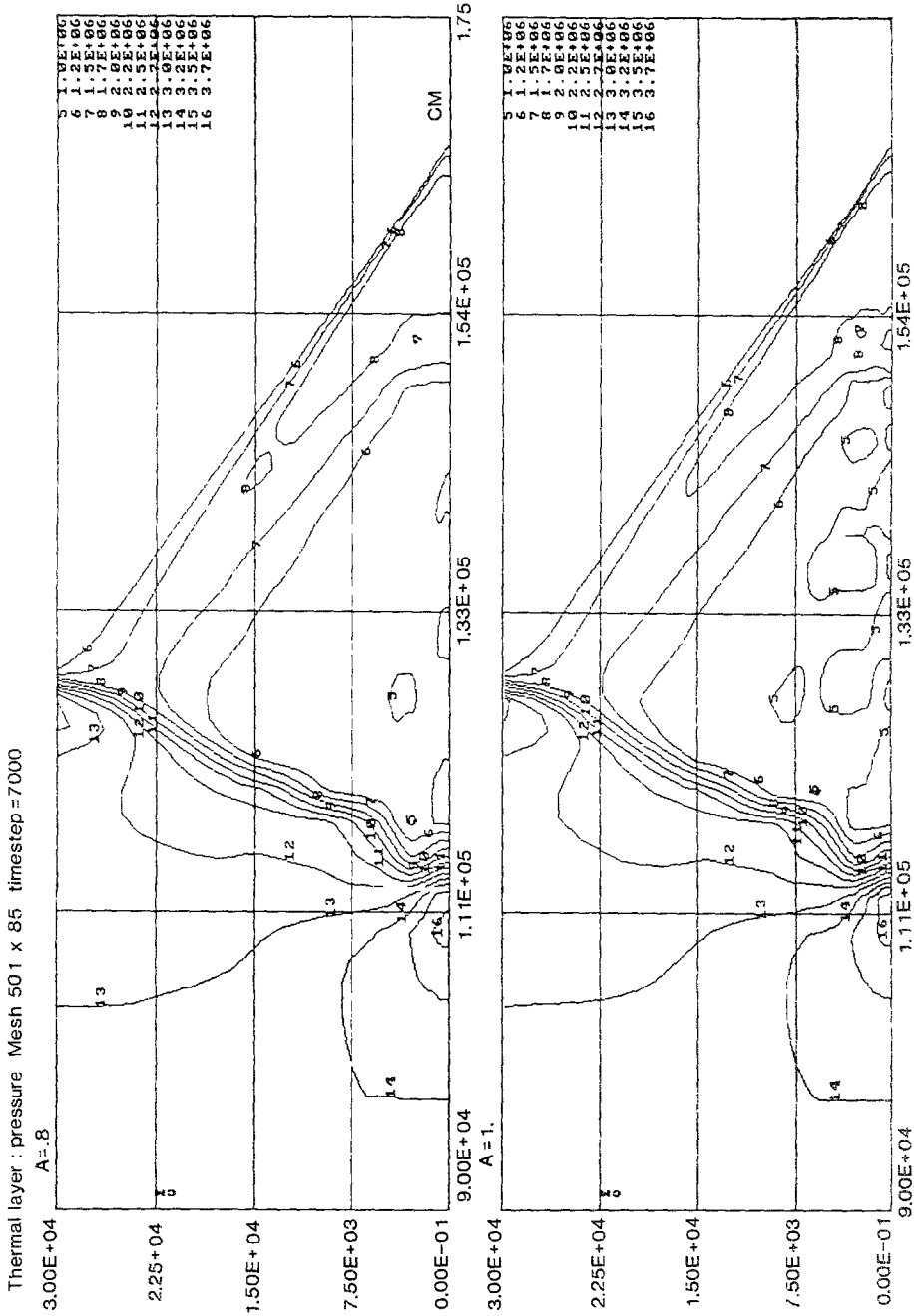
Thermal layer : pressure Mesh 501 x 85 timestep =7000
A=.8
A=1.
3.00E+04
2.25+04
1.50E+04
7.50E+03
0.00E-01
9.00E+04
1.11E+05
1.33E+05
1.54E+05
1.75
CM
5 1.0E+06
6 1.2E+06
7 1.5E+06
8 1.7E+06
9 2.0E+06
10 2.2E+06
11 2.5E+06
12 2.7E+06
13 3.0E+06
14 3.2E+06
15 3.5E+06
16 3.7E+06

FIGURE 7

The graph in Figs. 6 and 7 show the result for the problem of the thermal layer. A shock runs into a hot bottom layer which is a few meshpoints high. If one runs this problem for a long time in which the shock travels several hundred layer heights the MUSCL code did show instabilities in the layer. The graph shows two different calculations, namely one with $A = 0.8$ and a less diffusive one with $A = 1$. In the first case the occurrence of the instabilities are just beginning, where in the second case they are clearly shown. The fluctuations in the density are almost 15%. The code has not been run further, but it is clear that these instabilities will grow to the point of completely disrupting the computation. The MUSCL code achieves this earlier in time. It seems that this hydrodynamic instability is a very weak one and can be suppressed by a large enough numerical diffusion as illustrated in the comparison.

The graph in Fig. 8 shows a three-dimensional simulation of earth's bowshock (Ref. [7]). Intensive computations have been performed by Brecht *et al.* (Ref. [7]) of the interaction of the solar wind and the earth's magnetic field.

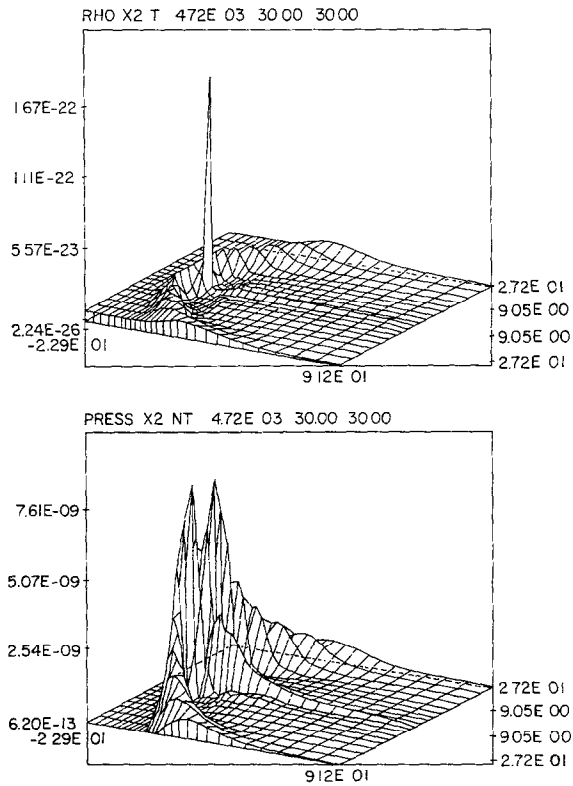

FIGURE 8

VI. Conclusion

The partial donor cell method (PDM) in conjunction with a central difference scheme has been used to simulate a variety of physical problems. The scheme is the simplest second-order scheme possible. As the central difference scheme is unstable, and the PDM operator suppresses these instabilities, it can resolve contact discontinuities quite well. The scheme is robust and fast.

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